THE OPTIMAL CONTROL OF GLOBAL WARMING

by

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1 A Non Technical Overview

This paper integrates a simple model of the global economy with a model of the global climate. This model is built with the purpose of calculating the business as usual path of greenhouse gas (GHG) emissions and the associated evolution of global warming over the very long term. The economic agents in this model are endowed with perfect foresight and behave in a manner consistent with private self interest. The model may also be used to determine the optimal control solution to the problem of climate change and to evaluate different emissions targets. This section contains a brief non technical overview of the model.

In the model economic output is a function of the inputs of land, labour, capital and fossil fuel. The three fossil fuels identified are conventional oil, coal and gas. The fossil fuel resources are finite. The quantity of land available is fixed but population follows a cumulative logistical growth path. Gross economic output is divided between consumption and investment goods, inter industry payments for energy and the costs of planting forests to soak up carbon from the atmosphere. The quantity of capital is determined by the amount of investment minus depreciation. Over time neutral technical progress increases the ease with which inputs can be transformed into output whilst climatic change has the opposite effect.

The extraction, distribution and consumption of fossil fuels results in the emission of GHGs. Other anthropogenic and non anthropogenic sources of GHGs are assumed to follow the rates of growth implicit in the Intergovernmental Panel on Climate Change (IPCC) projections. These emissions accumulate in the atmosphere and are removed only slowly. In the case of CO_2, the carbon cycle is represented by a certain fraction of emissions falling into one of five boxes. The total contents of these five boxes represents the airborne quantity of carbon as CO_2. These boxes leak carbon at different rates although one box does not leak at all implying that carbon emissions permanently elevate the concentration of CO_2 in the atmosphere. The removal of CH_4 and N_2O from the atmosphere is proportional to their atmospheric concentrations. Using the equations suggested by the IPCC the increase in radiative forcing from each GHG is calculated. The total change in radiative forcing is equal to the sum of the increases attributable to each gas. Equilibrium warming is equal to the change in radiative forcing multiplied by the climate sensitivity parameter and a climate feedback parameter. Actual warming adjusts to equilibrium warming via a partial adjustment mechanism.

In the absence of coercion the private incentives of individuals will be to ignore the impact on the climate of GHG emissions from fossil fuel use. Private individuals consume fossil fuel resources up to the point where the private present value marginal cost is equal to the marginal private benefit. The business as usual emissions trajectory is determined by equating the marginal private costs and benefits of resource utlisation and solving these simultaneously with the climate equations. This yields the unrestrained path of global warming.

With coercion the problem of climate change can be viewed as that of a social planner seeking to maximise intertemporal utility by allocating resources between competing ends. The maximisation problem implicitly determines the correct tax levels which must be placed upon the emission of each GHG in order to drive the economy along the path of maximum felicity. This tax must provide the incentives required by the owners of the fossil fuel resources to extract their resources at the rate identified by the social planner. The planner also has the option of sink enhancement to absorb carbon from the atmosphere. The scope of afforestation is however limited by the costs of drawing land from other productive uses and establishment cost. Furthermore, afforestation has only a delayed impact on the atmospheric concentration of CO_2 which soon expires. In the business as usual case no forests are planted since private individuals are unable to capture the benefit which they bestow. As is well known, provided the planner taxes all externalities at the appropriate rate the outcome is identical with that which would occur if a market existed for climate change.

A Comparison With Existing Cost Benefit Analyses Of Climate Change

The model presented here is similar to the CETA model of Peck and Tiesberg (1992) primarily in that carbon emissions from fossil fuels are determined by cost minimising substitution prompted by exhaustion induced price rises. However; the model presented here takes full advantage of the possibilities afforded by sink enhancement (ie planting forests) whereas CETA considers only the potential for reducing emissions. In a separate cost benefit analysis of arresting climate change Cline (1992) considers both emissions reductions and sink enhancement but arbitrarily limits both the scope and occasion of afforestation. Recently Nordhaus (1992) has developed an elegant optimal control model of GHG abatement entitled DICE. This model is similar to the one proposed below in that it represents an optimal growth model of the economy but differs crucially in that the emission of GHGs is in (declining) proportion of global output. In the model proposed below GHG emissions flow directly from the consumption of finite fossil fuel resources. The model therefore enables one to examine the reaction of the resource owners to any limits on carbon emissions which may be of some importance.

Finally, all cost benefit analyses of the global warming problem including this one have generally proceeded by replacing unknown parameters in their models with the expected values. The question they therefore address is what would be the optimal policy to follow if all the parameters of the model were known with perfect certainty. *Since it is absolutely not the case that all parameters are known with perfect certainty the results of these exercises are not policy relevant.* Usually it is assumed that the results of this exercise would not be significantly different. But nowhere has this been demonstrated.

2 A Detailed Description of the Model

The objective of the model is to maximise a discounted global utility function over a 200 year time horizon. This function is assumed to be time separable and contains the logarithm of per capita consumption multiplied by the population. A constant rate of utility discount (ρ) equal to 3% is applied over all time periods consistent with the historical savings ratio and the rates of return on capital.

$$Z = \sum_{t=1990}^{t=2190} [L_t \log (\frac{C_t}{L_t})](1+\rho)^{(t-1990)} \quad (1)$$

Gross output (Q_t) is divided into consumption (C_t), investment (I_t), costs of afforestation (A_t) and fossil fuel extraction costs (X_t):

$$Q_t = C_t + I_t + A_t + X_t \quad (2)$$

The production of gross output is modelled by a Cobb Douglas production function which exhibits constant returns to scale[2]. The production function contains labour (L_t), capital (K_t), land (H_t) and three fossil fuels consisting of conventional oil ($E1_t$), coal ($E2_t$) and gas ($E3_t$). These energy inputs are measured in exajoules (EJs). ω_1 to ω_6 are the respective shares in gross output. Neutral technical progress occurs at a rate of χ which is equal to an annual rate of 1.5% whilst (1-D_t) is a damage function (see below). Gross output is measured in 1990 $ trillions.

$$E_t = (1-D_t)\chi^{(t-1990)}\nu[\ L_t^{\omega_1}\ K_t^{\omega_2}\ H_t^{\omega_3}\ E1_t^{\omega_4}\ E2_t^{\omega_5}\ E3_t^{\omega_6}] \tag{3}$$

Global population is determined by the logistic function:

$$L_t = \frac{\pi_1}{1 + [\pi_2 \exp(-\pi_3 t)]} \tag{4}$$

π_1 represents the population ceiling which is taken as 11.31bn persons and corresponds to the World Bank estimate for 2100. The remaining parameters π_2 and π_3 are calibrated to the current population of 5.25bn and to the World Bank estimate for the year 2025 of 8.41bn. The next equation states that the end of period change in capital stock is equal to investment minus depreciation. Depreciation is assumed to be at exponential rate δ equal to 0.1 which corresponds to an average lifetime for capital of ten years.

$$K_{t+1} - K_t = I_t - \delta K_t \tag{5}$$

Consistent with the results of a survey by Cline (1992) the damage function assumes a loss of 1.1% of GNP for a 2.5°C temperature rise. D_t is the fraction of output lost as a result of climate change. Assuming that damage increases by a power of 1.3 with temperature rise ϵ_1 is 0.00334 and ϵ_2 is 1.3.

$$D_t = \epsilon_1 W_t^{\epsilon_2} \tag{6}$$

[2] A Cobb Douglas production function may be a poor representation of the substitution possibilities between fuels and hence the costs of abatement. This fact should be borne in mind whilst interpreting the results of the exercise. Later it may be possible to adopt a more flexible functional form.

The next equation deals with the cost of producing carbon based energy resources. Unit production costs (η) differ from unit price due to the exhaustion rent. In this model the determination of unit costs requires an iterative procedure. The unit costs of production are inferred by the requirement that the terminal conditions must be observed and that the business as usual solution must replicate the observed pattern of resource use in the base year[3]. Production costs might well be affected by the extent of depletion and rate of extraction but due to a lack of information it has been assumed that they do not.

$$X_t = \eta_1 E1_t + \eta_2 E2_t + \eta_3 E3_t \tag{7}$$

The next three identities relate changes in the stock of fossil fuels (S) to current period extraction:

$$S1_{t+1} - S1_t = -E1_t \tag{8}$$

$$S2_{t+1} - S2_t = -E2_t \tag{9}$$

$$S3_{t+1} - S3_t = -E3_t \tag{10}$$

The initial conditions on these stock variables denote the current availability of each type of fossil fuel:

$$S1_{t=1990} = 21250 \tag{11}$$

$$S2_{t=1990} = 271000 \tag{12}$$

$$S3_{t=1990} = 13950 \tag{13}$$

The following terminal conditions obviously apply:

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[3] Manne and Richels (1992) confront a similar problem in their well known GLOBAL 2100 model and deal with it in the same way.

$$S1_{t=2190} \geq 0 \tag{14}$$

$$S2_{t=2190} \geq 0 \tag{15}$$

$$S3_{t=2190} \geq 0 \tag{16}$$

The cost of afforestation in any time period depends upon the area of forest planted in that time period (P_t). The cost of plantation (γ) is set equal to \$0.0004trillion per mha (equivalent to \$400/ha).

$$A_t = \gamma P_t \tag{17}$$

The quantity of land available for productive purposes (as opposed to acting as a sink for carbon) evolves according to:

$$H_t = 500 - \sum_{t=1990}^{t} P_t \tag{18}$$

implying that up to 500mha are potentially available for afforestation worldwide. It is convenient to insist that the variable P_t be non negative. H_t is certainly always non negative. Hence:

$$P_t \geq 0 \tag{19}$$

$$H_t \geq 0 \tag{20}$$

The Emission Of GHGs

Turning attention to the physical environment this model deals in particular with each of the three main GHGs: CO_2, CH_4, and N_2O. CO_2 emissions from the consumption of fossil fuel are calculated by multiplying the consumption of each fuel by the respective carbon release coefficient ($\sigma_{1,1\text{-}3}$) and summing across all fuels. These carbon release coefficients are known to a high degree of accuracy. But CO_2 emissions arise also from the clearance of natural forests and to a much lesser extent from other industrial activities. The IPCC have projected the carbon emissions arising from each of these categories up to 2100. Autonomous emissions of CO_2 are forecast to decline from 1.5GtC per annum in 1990 to 0.5GtC per annum by 2100 as a result of a slackening in the rate of tropical deforestation. These estimates suggest an average decline of about 1% per annum[4].

As previously explained the plantation of new forests has the effect of reducing atmospheric stocks of CO_2. The total quantity of carbon stored in this fashion, net of that contained in the vegetation which the forest replaces, is assumed to be 0.18GtC/mha (180tC/ha). However; the impact of afforestation is exhausted after 40 years have elapsed since by then the trees are fully grown. The average annual quantity of carbon sequestered by forests (θ) is thus 0.0045GtC/mha up to 40 years after plantation and zero thereafter. Net flows of carbon into the atmosphere ($N1_t$) are thus the sum of fossil fuel and autonomous emissions minus carbon sequestered by the forests:

$$NI_t = \sigma_{1,1}EI_t + \sigma_{1,2}E2_t + \sigma_{1,3}E3_t + \beta_1 e^{\phi_1(t-1990)} - \theta \sum_{t-40}^{t} P_t \qquad t \geq 2030 \tag{21}$$

and:

$$NI_t = \sigma_{1,1}EI_t + \sigma_{1,2}E2_t + \sigma_{1,3}E3_t + \beta_1 e^{\phi_1(t-1990)} - \theta \sum_{t=1990}^{t} P_t \qquad t < 2030 \tag{22}$$

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[4] It seems paradoxical to consider the use of afforestation to soak up carbon from the atmosphere whilst allowing tropical deforestation to continue. If it is assumed that all deforestation can be costlessly halted by institutional reform then autonomous carbon emissions under the IPCC rise from 0.2GtC to 0.6GtC by 2100; a growth rate of 1%.

In the case of CH_4 around 20% of emissions in 1990 were connected with the extraction, distribution and consumption of fossil fuel. These may be (very approximately) apportioned between the different fossil fuels in order to determine the fossil fuel release coefficients per EJ. The expected growth rate ϕ_2 of autonomous emissions up to 2100 is 0.47% per annum and the flow of non energy emissions in 1990 (β_2) 415Tg. Total emission of CH_4 ($N2_t$) is given by:

$$N2_t = \sigma_{2,1}E1_t + \sigma_{2,2}E2_t + \sigma_{2,3}E3_t + \beta_2 e^{\phi_2(t-1990)} \tag{23}$$

In 1990 0.4TgN were associated with fossil fuel use. These cannot be apportioned between different fuels so they are apportioned on an energy supplied basis. The rate of growth ϕ_3 of autonomous emissions of N_2O up to the year 2100 is projected to be 0.24%. 1990 autonomous emissions (β_3) amounted to 12.5TgN. Total emissions are:

$$N3_t = \sigma_{3,1}E1_t + \sigma_{3,2}E2_t + \sigma_{3,3}E3_t + \beta_3 e^{\phi_3(t-1990)} \tag{24}$$

The Atmosphere

The dynamics of CO_2 in the atmosphere are governed by the carbon cycle. The model of the carbon cycle used here is that of Maier-Reimer and Hasselman (1987). The MRH model treats CO_2 as falling into one of 5 different boxes (B). The CO_2 in each of these boxes possesses a different atmospheric lifetime[5]. The atmospheric concentration of CO_2 is proportionate to the summed contents of all five boxes. The fractions (α_b) of new carbon emissions falling into each box are 0.13, 0.20, 0.32, 0.25 and 0.10. The associated lifetimes ($\tau_{1,b}$) are ∞, 363, 74, 17 and 2 years respectively. Hence:

$$B_{b,t+1} - B_{b,t} = \alpha_b N1_t - \frac{B_{b,t}}{\tau_{1,b}} \qquad \forall b \tag{25}$$

The initial contents of each box measured in GtC is given by:

The concentration of CO_2 in the atmosphere ($M1_t$) measured in units of ppm is given by:

[5] The residence times of different gases are defined to be equal to the ratio between the atmospheric content of the a gas and the total rate of removal. CO_2 is a special case since it is not destroyed but rather circulated between various reservoirs.

$$B_{b,t=1990} = \bar{B}_b \tag{26}$$

$$M1_t = \mu_1 \sum_{b=1}^{b=5} B_{b,t} \tag{27}$$

where μ_1=0.471ppm of CO_2 per GtC (see IPCC, 1990).

The end of period change in the atmospheric concentrations of CH_4 and N_2O depends only upon their current atmospheric concentration, the average residence times of the different gases (τ) and also upon the quantity of each gas released during that period.

$$M2_{t+1} - M2_t = \mu_2 N2_t - \frac{M2_t}{\tau_2} \tag{28}$$

$$M3_{t+1} - M3_t = \mu_3 N3_t - \frac{M3_t}{\tau_3} \tag{29}$$

Atmospheric lifetimes for CH_4 and N_2O are 11 years and 130 years respectively (IPCC 1992). The coefficients μ_2 and μ_3 represent the increase in atmospheric concentration of the respective gases per unit released. The atmospheric concentration of CH_4 and N_2O are measured in parts per billion by volume (ppbv). 1TgCH_4 corresponds to 0.351 ppbv and 1TgN to 0.207 ppbv N_2O (IPCC 1990). These define the initial states:

$$M2_{t=1990} = 1720 \tag{30}$$

$$M3_{t=1990} = 310 \tag{31}$$

In order to calculate the change in global temperature brought about by an elevated concentration of GHGs in the atmosphere it is necessary to calculate the change in radiative forcing attributable to each gas. Radiative forcing rises less than linearly with concentrations since some spectral bands become effectively saturated. The following functional forms suggested by Dickinson and Cicerone (1986) and Wigley (1987) are adopted.

$$R1_t = \psi_1(\log(M1_t) - \log(280)) \quad (32)$$

$$R2_t = \psi_2(\sqrt{M2_t} - \sqrt{800}) \quad (33)$$

$$R3_t = \psi_3(\sqrt{M3_t} - \sqrt{285}) \quad (34)$$

Parameters ψ_1=6.3, ψ_2=0.036, and ψ_3=0.14 (IPCC 1990). The combined radiative forcing of all the other myriad GHGs is modelled as growing exponentially at rate of ϕ_4=0.012 implicit in the IPCC (1990) BAU scenario. Radiative forcing from this source (β_4) in 1990 was estimated to be 0.287 wm^{-2}.

$$R_{4,t} = \beta_4 e^{\phi_4(t-1990)} \quad (35)$$

The changes in radiative forcing attributable to each of the different gases are added to find total change in radiative forcing R_t[6].

$$R_t = R1_t + R2_t + R3_t + R4_t \quad (36)$$

In order to calculate the equilibrium temperature rise from the change in radiative forcing it is necessary to multiply the change in radiative forcing by the climate sensitivity parameter measured in $Kw^{-1}m^2$ and a dimensionless feedback parameter[7]. Taking the IPCC central estimate of 2.5°C warming for CO_2 doubling it is possible to calculate that λ, the product

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[6] Radiative forcing is strictly less than additive since significant spectral overlapping occurs between CH_4 and N_2O. See IPCC 1992 for details.

[7] For an interpretation of the feedback parameter see IPCC 1990.

of the feedback and sensitivity parameters is $0.572Kw^{-1}m^2$ [8]. Given that the climate sensitivity parameter is widely agreed to be $0.3Kw^{-1}m^2$ the feedback parameter is 1.91. However, the feedback parameter could easily be as high as 3.4 or as low as 1.1. Equilibrium warming U_t is given by:

$$U_t = \lambda R_t \tag{37}$$

The change in current temperature is a lagged function of equilibrium warming and current warming due to the thermal inertia of the oceans. ξ is the time delay parameter whose value is estimated to be 0.027 (see Manabe and Stouffer, 1987).

$$W_{t+1} - W_t = \zeta_0(U_t - W_t) \tag{38}$$

Current observed warming above pre industrial levels of 0.45°C defines the initial condition for W_t.

$$W_{t=1990} = 0.45$$

This concludes the specification of the model.

3 Shadow Values

The shadow values on each of these equations readily lend themselves to economic interpretation although not all are of equal interest. The shadow value of the income identity is interpretable as the present value of an additional $1 trillion in terms of utility. In order to calculate the current value it is necessary to divide this value by the utility discount factor. The shadow values associated with the equations determining the release of GHGs into the atmosphere multiplied by the current value marginal utility of money yields the current value

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$$U_t = \lambda RI_t$$

Substituting Wigley's equation for the change in direct forcing from CO_2 and doubling the concentration of carbon dioxide gives:

$$\lambda = \frac{2.5}{6.3\log(2)} = 0.572$$

of the damage caused by releasing an additional unit of gas. This value is synonomous with the optimal tax rate measured in units of $trillion per GtC, per TgCH4 and per TgN. Multiplying the first of these figures by 1000 provides the tax in terms of $/tC. Multiplying the latter two values by $1x10^6$ yields the tax rate in terms of $/tCH4 and $/tN respectively.

The terminal date in this model (2190) is sufficiently distant such that projecting the model further into the future does not significantly alter the results over the periods reported.

4 Results

Preliminary results of the model will be presented at the workshop.